fourth GRADE

Decimal Fractions & Geometry

MATH IN FOCUS

Unit 3 Curriculum Guide

February 4th, 2019 – April 18th, 2019

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Unit Overview

Unit 3: Chapters 12, 9, 10 **Eureka Module 5:** Fractions Equivalence, Ordering, and Operations (TOPICS B, C, D ONLY)
In this Unit Students will:
• Measure and draw angles and solve problems involving angle measures. • Identify and draw perpendicular and parallel line segments as well as horizontal and vertical lines. • Identify squares and rectangles based on their properties, and find unknown angle measures and side lengths of figures. • Measure time to the minute, second, and hour • Use metric units of length, mass, and volume to solve real-world measurement problems. • Use customary units of length, weight, and capacity to solve- real world measurement problem.
Essential Questions
➢ What geometric terms describe types of angles? ➢ How do you find the measure of an angle using equivalent fractions? ➢ How are angles measured? ➢ How can you draw an angle? ➢ How can you add and subtract to find unknown angle measurements? ➢ What are some important geometric names for lines? ➢ How can you identify polygons? ➢ How can you classify triangles? ➢ How can you classify quadrilateral? ➢ How can perimeter and area formulas be used to solve problems? ➢ How can you measure and find the area of a rectangle by multiplying? ➢ How can you measure and find the area of a rectangle by using a formula? ➢ How do you estimate and measure length? ➢ How do you measure capacity with customary units? ➢ How do you measure weight? ➢ How do you change customary units? ➢ How do you estimate and measure length using metric units? ➢ How do you measure capacity with metric units? ➢ How do you measure mass? ➢ How do you change metric units? ➢ How do you compare units of time? ➢ How can you work backward to solve a problem?

Enduring Understandings
• Chapter 9: Angles ✓ Estimate and measure angles ✓ Use a protractor to measure angels ✓ Identify acute, obtuse and right angles ✓ Find unknown angle measurements ✓ Solve real-world problems by finding unknown angles measures • Chapter 10: Perpendicular and Parallel Line Segments ✓ Draw perpendicular line segments ✓ Draw parallel line segments ✓ Identify horizontal and vertical lines • Chapter 12: Conversion of Measurements ✓ Understand relative sizes of measurement units ✓ Convert metric units of length, mass, and volume ✓ Convert customary units of length, weight, and volume ✓ Convert units of time ✓ Use four operations to solve word problems

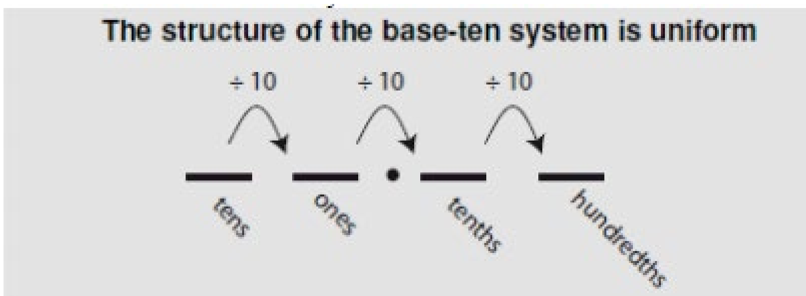
MIF Pacing Guide

Eureka Math Module 6 (TOPICS C,D,E) & MIF Chapter 12,9,10		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic C: Decimal Comparison	Lesson 9	Use the place value chart and metric measurement to compare decimals and answer comparison questions. https://www.youtube.com/watch?v
	Lesson 10	Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =. https://www.youtube.com/watch?v
	Lesson 11	Compare and order mixed numbers in various forms. https://www.youtube.com/watch?v
Topic D: Addition with Tenths and Hundredths	Lesson 12	Apply understanding of fraction equivalence to add tenths and hundredths. https://www.youtube.com/watch?v
	Lesson 13	Add decimal numbers by converting to fraction form. https://www.youtube.com/watch?v
	Lesson 14	Solve word problems involving the addition of measurements in decimal form. https://www.youtube.com/watch?v
Topic E: Money Amounts as Decimal Numbers	Lesson 15	Express money amounts given in various forms as decimal numbers. https://www.youtube.com/watch?v
	Lesson 16	Solve word problems involving money. https://www.youtube.com/watch?v

MIF Chapter 12,9,10		
Activity	**Common Core Standards**	**Estimated Time (# of block)**
12.1 Length Day 1	4.MD.1	1 Block
12.1 Length Day 2	4.MD.1	1 Block
12.1 Length Day 3	4.MD.1	1 Block
12.2 Mass, Weight, and Volume Day 1	4.MD.1	1 Block
12.2 Mass, Weight, and Volume Day 2	4.MD.1	1 Block
12.3 Time	4.MD.1	1 Block
12.4 Real- World Problems: Measurement Day 1	4.MD.1	1 Block
12.4 Real- World Problems: Measurement Day 2	4.MD.1	1 Block
Chapter 9 Opener	4.MD.5, 4.G.1	1 Block
9.1 Understanding and Measuring Angles Day 1	4.G.1 4.MD.5-6	1 Block
9.1 Understanding and Measuring Angles Day 2	4.G.1 4.MD.5-6	1 Block
9.2 Drawing Angles to 180° Day 1	4.G.1 4. MD.6	1 Block
9.2 Drawing Angles to 180° Day 2	4.G.1 4. MD.6	1 Block
9.3 Turns and Angle Measures Day 1	4.MD.5, 7	1 Block
9.3 Turns and Angle Measures Day 2	4.MD.5, 7	1 Block
Problem Solving	4.MD.5	1 Block
Chapter 10 Opener	4.G.1	1 Block
10.1 Drawing Perpendicular Line Segments Day 1	4.G.1-2	1 Block
10.1 Drawing Perpendicular Line Segments Day 2	4.G.1-2	1 Block
10.2 Drawing Parallel Line Segments Day 1	4.G.1-2	1 Block
10.2 Drawing Parallel Line Segments Day 2	4.G.1-2	1 Block
10.3 Horizontal and Vertical Lines	4.G.1	1 Block
Problem Solving	4.G.1-2	1 Block

Common Core State Standards	
4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

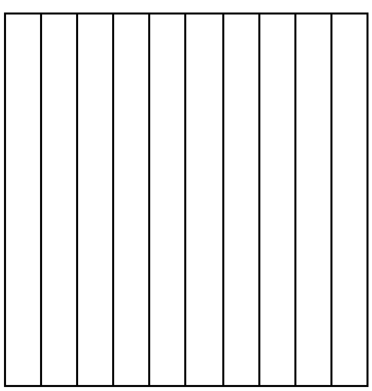
This standard continues the work of equivalent fractions by having students change fractions with a 10 in the
denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with
decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support
this work. Student experiences should focus on working with grids rather than algorithms.
Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.
Students in fourth grade work with fractions having denominators 10 and 100. Because it involves partitioning
into 10 equal parts and treating the parts as numbers called one tenth and one hundredth, work with these fractions can be used as preparation to extend the base-ten system to non-whole numbers.



This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade. Example:

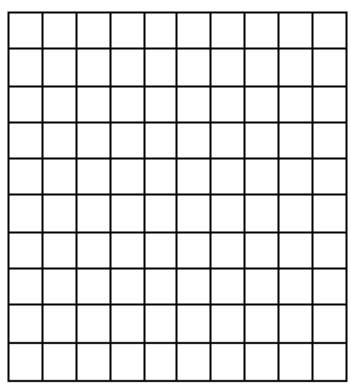
Ones	.	Tenths	Hundredths

Tenths Grid



.3 = 3 tenths = 3/10

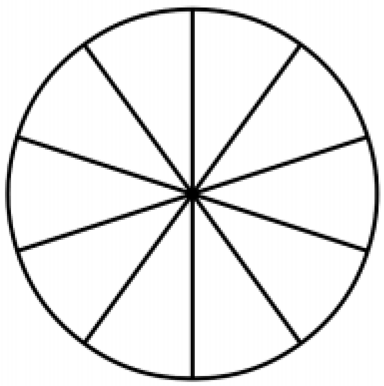
Hundredths Grid



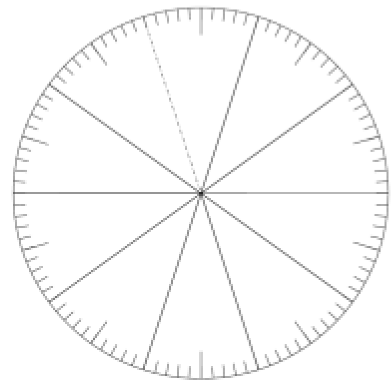
.30 = 30 hundredths = 30/100

Example: Represent 3 tenths and 30 hundredths on the models below.

10ths circle



100ths circle



4.NF.6

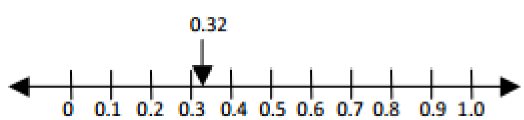
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4 ,5, 6, 8, 10, 12, and 100.

Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal. Students make connections between fractions with denominators of 10 and 100 and the place value chart. By
reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	•	Tenths	Hundredths
			•	3	2

Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.



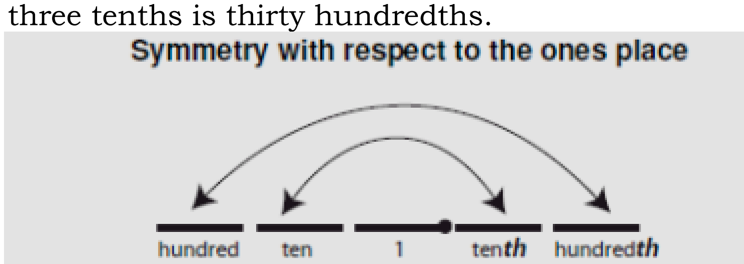
4.NF.7

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

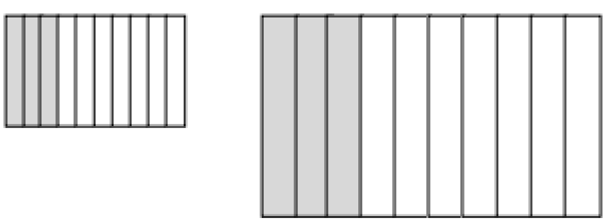
Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.
The decimal point is used to signify the location of the ones place, but its location may suggest there should be a“oneths" place to its right in order to create symmetry with respect to the

decimal point. However, because one is the basic unit from which the other base ten units are derived, the symmetry occurs instead with respect to the ones place. Ways of reading decimals aloud vary. Mathematicians and scientists often read 0.15 aloud as “zero point one five" or “point one five." (Decimals smaller than one may be written with or without a zero before the decimal point.)Decimals with many non-zero digits are more easily read aloud in this manner. (For example, the number π, which has infinitely many non-zero digits, begins 3.1415 . ..)

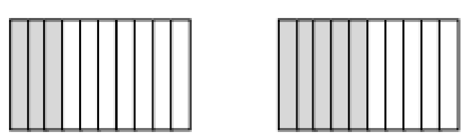
Other ways to read 0.15 aloud are “1 tenth and 5 hundredths” and “15 hundredths,” just as 1,500 is sometimes read “15 hundred” or “1 thousand, 5 hundred.” Similarly, 150 is read “one hundred and fifty” or “a hundred fifty” and understood as 15 tens, as 10 tens and 5 tens, and as 100 + 50. Just as 15 is understood as 15 ones and as 1 ten and 5 ones in computations with whole numbers, 0.15 is viewed as 15 hundredths and as 1 tenth and 5 hundredths in computations with decimals. It takes time to develop understanding and fluency with the different forms. Layered cards for decimals can help students become fluent with decimal equivalencies such as three tenths is thirty hundredths.



Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.



When the wholes are the same, the decimals or fractions can be compared. Example: Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.



4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, millimeter, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students develop benchmarks and mental images about a meter (e.g., about the height of a tall chair) and a kilometer (e.g., the length of 10 football fields including the end zones, or the distance a person might walk in about 12 minutes), and they also understand that "kilo" means a thousand, so 3000 m is equivalent to 3 km.

Expressing larger measurements in smaller units within the metric system is an opportunity to reinforce notions of place value. There are prefixes for multiples of the basic unit (meter or gram), although only a few (kilo-, centi-, and milli-) are in common use. Tables such as the one below are an opportunity to develop or reinforce place value concepts and skills in measurement activities. Relating units within the metric system is another opportunity to think about place value. For example, students might make a table that shows measurements of the same lengths in centimeters and meters. Relating units within the traditional system provides an opportunity to engage in mathematical practices, especially "look for and make use of structure" and "look for and express regularity in repeated reasoning" For example, students might make a table that shows measurements of the same lengths in feet and inches.

Super- or subordinate unit	Length in terms of basic unit
kilometer	10^3 or 1000 meters
hectometer	10^2 or 100 meters
decameter	10^1 or 10 meters
meter	1 meter
decimeter	10^{-1} or $\frac{1}{10}$ meters
centimeter	10^{-2} or $\frac{1}{100}$ meters
millimeter	10^{-3} or $\frac{1}{1000}$ meters

Centimeter and meter equivalences

cm	m
100	1
200	2
300	3
500	
1000	

Foot and inch equivalences

feet	inches
0	0
1	12
2	24
3	

Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12. Example: Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	***n*** x 3

Foundational understandings to help with measure concepts:
Understand that larger units can be subdivided into equivalent units (partition).
Understand that the same unit can be repeated to determine the measure (iteration).
Understand the relationship between the size of a unit and the number of units needed (compensatory principal).
These Standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an
object. Weight is the force exerted on the body by gravity. On the earth's surface, the distinction is not important (on the moon, an object would have the same mass, would weigh less due to the lower gravity).

4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
4.MD.5a	An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
4.MD.5b	An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

- This standard brings up a connection between angles and circular measurement (360 degrees). Angle measure is a "turning point" in the study of geometry. Students often find angles and angle measure to be difficult concepts to learn, but that learning allows them to engage in interesting and important mathematics. An angle is the union of two rays, **a** and **b**, with the same initial point P. The rays can be made to coincide by rotating one to the other about P; this rotation determines the size of the angle between **a** and **b**. The rays are sometimes called the sides of the angles.

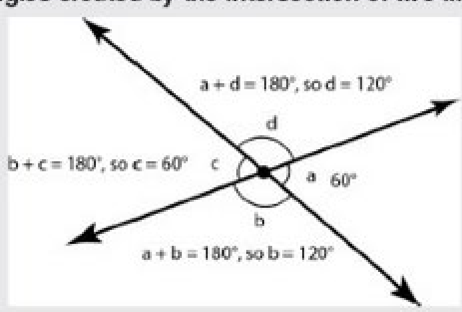
- Another way of saying this is that each ray determines a direction and the angle size measures the change from one direction to the other. Angles are measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and degrees are the unit used to measure angles in elementary school. A full rotation is thus 360°

- Two angles are called complementary if their measurements have the sum of 90°. Two angles are called supplementary if their measurements have the sum of 180°. Two angles with the same vertex that overlap only at a boundary (i.e., share a side) are called adjacent angles. These terms may come up in classroom discussion, they will not be tested. This concept is developed thoroughly in middle school (7th grade).
- Like length, area, and volume, angle measure is additive: The sum of the measurements of adjacent angles is the measurement of the angle formed by their union. This leads to other important properties. If a right angle is decomposed into two adjacent angles, the sum is 90°, thus they are complementary. Two adjacent angles that compose a “straight angle” of 180° must be supplementary.

An angle

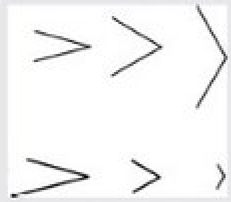
name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

Angles created by the intersection of two lines



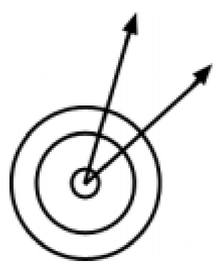
When two lines intersect, they form four angles. If the measurement of one is known (e.g., angle *a* is 60°), the measurement of the other three can be determined.

Two representations of three angles



Initially, some students may correctly compare angle sizes only if all the line segments are the same length (as shown in the top row). If the lengths of the line segments are different (as shown in the bottom row), these students base their judgments on the lengths of the segments, the distances between their endpoints, or even the area of the triangles determined by the drawn arms. They believe that the angles in the bottom row decrease in size from left to right, although they have, respectively, the same angle measurements as those in the top row.

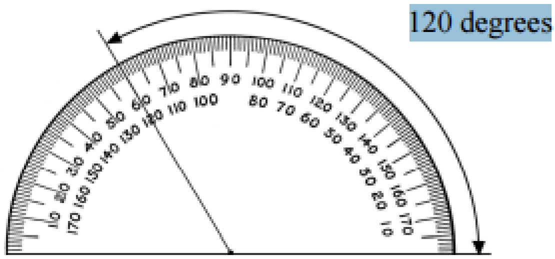
- The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.

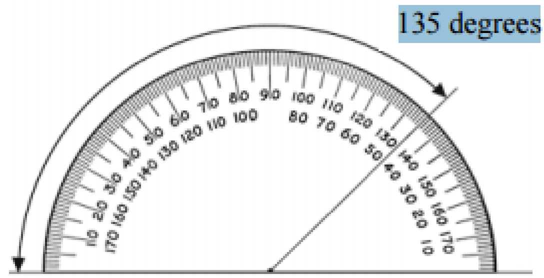


This standard calls for students to explore an angle as a series of “one-degree turns.” A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100°, how many one-degree turns has the sprinkler made?

4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

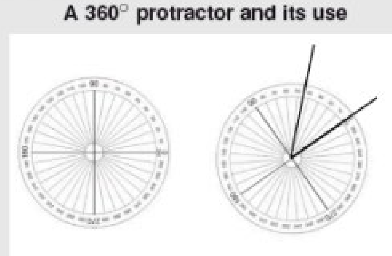
- Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360° rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90° and 180°. They extend this understanding and recognize and sketch angles that measure approximately 45° and 30°.
- They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular). Students should measure angles and sketch angles.





- As with all measureable attributes, students must first recognize the attribute of angle measure, and distinguish it from other attributes!
- As with other concepts students need varied examples and explicit discussions to avoid learning limited ideas about measuring angles (e.g., misconceptions that a right angle is an angle that points to the right, or two right angles represented with different orientations are not equal in measure).
  - If examples and tasks are not varied, students can develop incomplete and inaccurate notions. For example, some come to associate all slanted lines with 45° measures and horizontal and vertical lines with measures of 90°. Others believe angles can be "read off" a protractor in "standard" position, that is, a base is horizontal, even if neither ray of the angle is horizontal. Measuring and then sketching many angles with no horizontal or vertical ray perhaps initially using circular 360° protractors can help students avoid such limited conceptions.

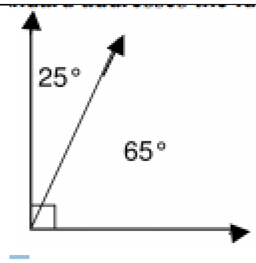
A 360° protractor and its use



The figure on the right shows a protractor being used to measure a 45° angle. The protractor is placed so that one side of the angle lies on the line corresponding to 0° on the protractor and the other side of the angle is located by a clockwise rotation from that line.

4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

- This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.

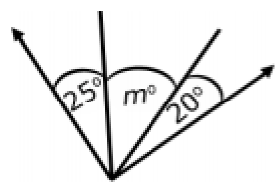


Example:

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved? If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:

If the two rays are perpendicular, what is the value of m?



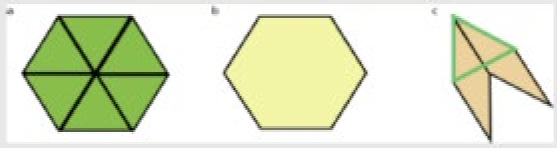
Example:

Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30°. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

- Students can develop more accurate and useful angle and angle measure concepts if presented with angles in a variety of situations. They learn to find the common features of superficially different situations such as turns in navigation, slopes, bends, corners, and openings.
- With guidance, they learn to represent an angle in any of these contexts as two rays, even when both rays are not explicitly represented in the context; for example, the horizontal or vertical in situations that involve slope (e.g., roads or ramps), or the angle determined by looking up from the horizon to a tree or mountain-top. Eventually they abstract the common attributes of the situations as angles (which are represented with rays and a

vertex,) and angle measurements.

Determining angles in pattern blocks

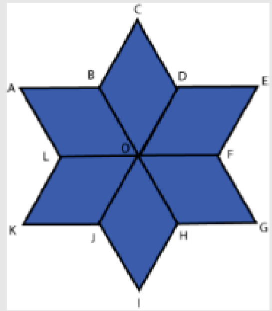


Students might determine all the angles in the common "pattern block" shape set based on equilateral triangles. Placing six equilateral triangles so that they share a common vertex (as shown in part a), students can figure out that because the sum of the angles at this vertex is 360°, each angle which shares this vertex must have measure 60°. Because they are congruent, all the angles of the equilateral triangles must have measure 60° (again, to ensure they develop a firm foundation, students can verify these for themselves with a protractor). Because each angle of the regular hexagon (part b) is composed of two angles from equilateral triangles, the hexagon's angles each measure 120°. Similarly, in a pattern block set, two of the smaller angles from tan rhombi compose an equilateral triangle's angle, so each of the smaller rhombus angles has measure 30°.

- Students with an accurate conception of angle can recognize that angle measure is additive. As with length, area, and volume, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students can then solve interesting and challenging addition and subtraction problems to find the measurements of unknown angles on a diagram in real world and mathematical problems.

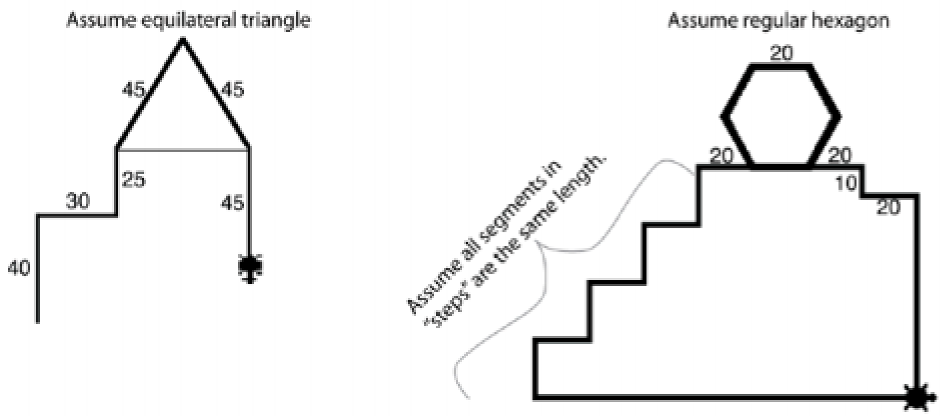
- For example, they can find the measurements of angles formed by a pair of intersecting lines, as illustrated above, or given a diagram showing the measurement of one angle, find the measurement of its complement. They can use a protractor to check measurement, not to check their reasoning, but to ensure that they develop full understanding of the mathematics and mental images for important benchmark angles (e.g., 30°, 45°, 60°, and 90°).

Determining angle measurements



Students might be asked to determine the measurements of the following angles:
$\angle BOD$
$\angle BOF$
$\angle ODE$
$\angle CDE$
$\angle CDJ$
$\angle BHG$

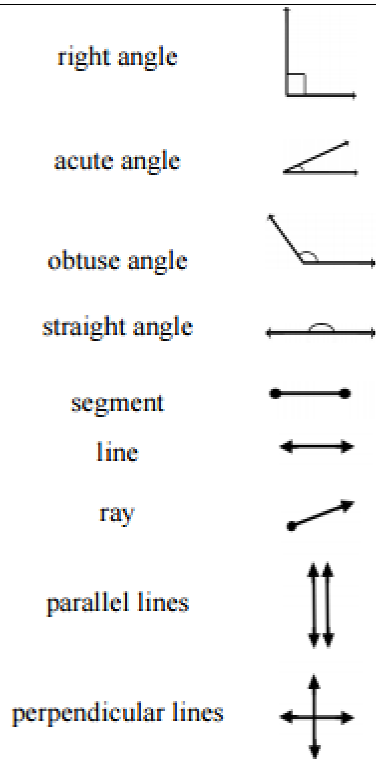
(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 24)



(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 25)

4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

- This standard asks students to draw two-dimensional geometric objects and to also identify them in two dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students may not easily identify lines and rays because they are more abstract.



- Developing a clear understanding that a point, line, and plane are the core attributes of space objects, and real world situations can be used to think about these attributes. Enforcing precise geometrical vocabulary is important for mathematical communication.

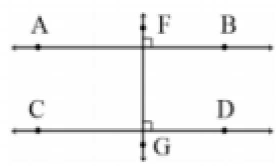
Example:

How many acute, obtuse and right angles are in this shape?

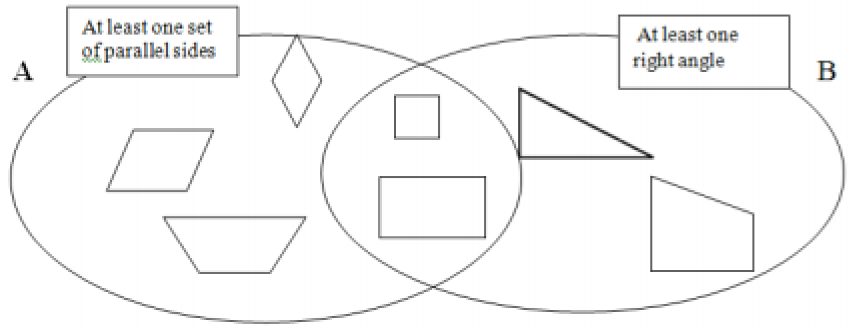
- Line segments and rays are sets of points that describe parts of lines, shapes, and solids. Angles are formed by two intersecting lines or by rays with a common endpoint. They are classified by size.

4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

- Classify triangles based on the presence or absence of perpendicular lines and based on the presence or absence of angles of a particular size.
- Classify quadrilaterals based on the presence or absence of parallel or perpendicular lines and based on the presence or absence of angles of a particular size.
- Two-dimensional or plane shapes have many properties that make them different from one another. Students should become familiar with the concept of parallel and perpendicular lines.
- Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90º). Parallel and perpendicular lines are shown below:



- Polygons can be described and classified by their sides and angles. Identify triangles, quadrilaterals, pentagons, hexagons, and octagons based on their attributes. Have a clear understanding of how to define and identify a right triangle.
- Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles. A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other.
- This standard calls for students to sort objects based on parallelism, perpendicularity and angle types. Example: Which figure in the Venn diagram below is in the wrong place, explain how do you know?



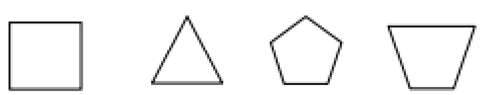
- Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles. Example: Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:

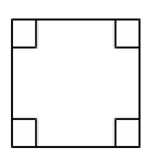
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counter example.

- A parallelogram with exactly one right angle.
- An isosceles right triangle.
- A rectangle that is not a parallelogram. (impossible)
- Every square is a quadrilateral.
- Every trapezoid is a parallelogram.

Example: Identify which of these shapes have perpendicular or parallel sides and justify your selection.



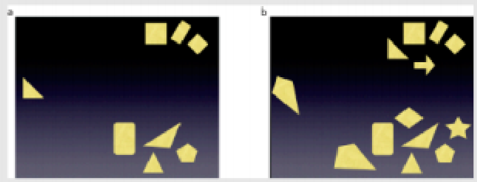
A possible justification that students might give is: The square has perpendicular lines because the sides meet at a corner, forming right angles



- Angle Measurement: This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1. Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.

- Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides.

Guess My Rule



Students can be shown the two groups of shapes in part a and asked "Where does the shape on the left belong?" They might surmise that it belongs with the other triangles at the bottom. When the teacher moves it to the top, students must search for a different rule that fits all the cases.

Later (part b), students may induce the rule: "Shapes with at least one right angle are at the top." Students with rich visual images of right angles and good visualization skills would conclude that the shape at the left (even though it looks vaguely like another one already at the bottom) has one right angle, thus belongs at the top.

- The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do not appear until middle school

- **TEACHER NOTE:** In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, a parallelogram is not a trapezoid. (Progressions for the CCSSM: Geometry, June 2012.)

M : Major Content | S: Supporting Content | A : Additional Content

MIF Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	**Chapter Opener** Assessing Prior Knowledge *The Pre Test serves as a diagnostic test of readiness of the upcoming chapter*	**Teacher Materials** Quick Check Pretest (Assessm't Bk) Recall Prior Knowledge **Student Materials** Student Book (Quick Check); Copy of the Pre Test; Recall prior Knowledge	Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test ; Students write Quick Check answers on a separate sheet of paper Quick Check and the Pre Test can be done in the same block *(See Anecdotal Checklist; Transition Guide)* Recall Prior Knowledge – Quick Check – Pre Test
DIRECT ENGAGEMENT	**Direct Involvement/Engagement** Teach/Learn *Students are directly involved in making sense, themselves, of the concepts – by interacting the tools, manipulatives, each other, and the questions*	**Teacher Edition** 5-minute warm up Teach; Anchor Task **Technology** Digi **Other** Fluency Practice	• The Warm Up activates prior knowledge for each new lesson • Student Books are CLOSED; Big Book is used in Gr. K • Teacher led; Whole group • Students use concrete manipulatives to explore concepts • A few select parts of the task are explicitly shown, but the majority is addressed through the hands-on, constructivist approach and questioning • Teacher facilitates; Students find the solution
GUIDED LEARNING	**Guided Learning and Practice** Guided Learning	**Teacher Edition** Learn **Technology** Digi **Student Book** Guided Learning Pages Hands-on Activity	Students-already in pairs /small, homogenous ability groups; Teacher circulates between groups; Teacher, anecdotally, captures student thinking **Small Group w/Teacher circulating among groups** Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time

INDEPENDENT PRACTICE	**Independent Practice** *A formal formative assessment*	**Teacher Edition** Let's Practice **Student Book** Let's Practice **Differentiation Options** All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment	**Let's Practice** determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice. Manipulatives **CAN** be used as a communications tool as needed. Completely Independent On level/advance learners should finish all workbook pages.
ADDITIONAL PRACTICE	**Extending the Lesson**	Math Journal Problem of the Lesson Interactivities Games	
	Lesson Wrap Up	Problem of the Lesson Homework (Workbook , Reteach, or Extra Practice)	Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice) Reteach Homework (issued to struggling learners) should be checked the next day
POST TEST	**End of Chapter Wrap Up and Post Test**	**Teacher Edition** Chapter Review/Test Put on Your Thinking Cap **Student Workbook** Put on Your Thinking Cap **Assessment Book** Test Prep	Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is graded/scored. The Chapter Review/Test can be completed • Individually (e.g. for homework) then reviewed in class • As a 'mock test' done in class and doesn't count • As a formal, in class review where teacher walks students through the questions Test Prep is completely independent; scored/graded Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own.

Unit 2 Math Background

During their elementary mathematics education, students were exposed to the following:

- A point, line, and line segment
- Estimated size of angles by comparing them to right angles
- Draw perpendicular lines using a square grid
- Recognize perpendicular and parallel lines
- Use protractor to draw triangles, perpendicular line segments
- Describe two-dimensional shapes by their sides and angles
- Measure length, mass, or weight, and volume in metric units
- Read and tell time to the minute
- Use the dollar sign and decimal point in money amounts

Potential Student Misconceptions

Decimal Fractions

- Students may be confused with what part of a whole a decimal represents. When working with decimal numbers less that one whole, a foundational understanding that needs to be developed is 0.01 represents one out of 100 parts of the whole, and it is one of 10 parts of a tenth (a tenth of a tenth). Students need activities using concrete models to understand this concept.
- Watch for students who think that 0.54 is greater 54 is greater than 8. These students do not understand the relationship between tenths and hundredths and need more experience with modeling decimals. Key to their understanding is the fact that 0.8 is equivalent to 0.80.

Chapter 9

- Students may struggle with naming angles. Draw an angle on the board and label it.
- Students are confused to which number to use when determining the measure of an angle using a protractor.
- Students should compare angles to the benchmark of 90 degrees or right angles.
- Students may not use their angle strips correctly.

Chapter 10

- Students may not find 90 degree on their protractor, so the line they draw will not be perpendicular.
- Students may have difficulty setting up their drawing triangle and straightedge. Remind the students to place the drawing triangle with the right angle on the line segment and the straightedge along the base of the drawing triangle.
- Students confuse the term horizontal and vertical. The sun setting is on the horizon to remember horizontal vs vertical.

Chapter 12

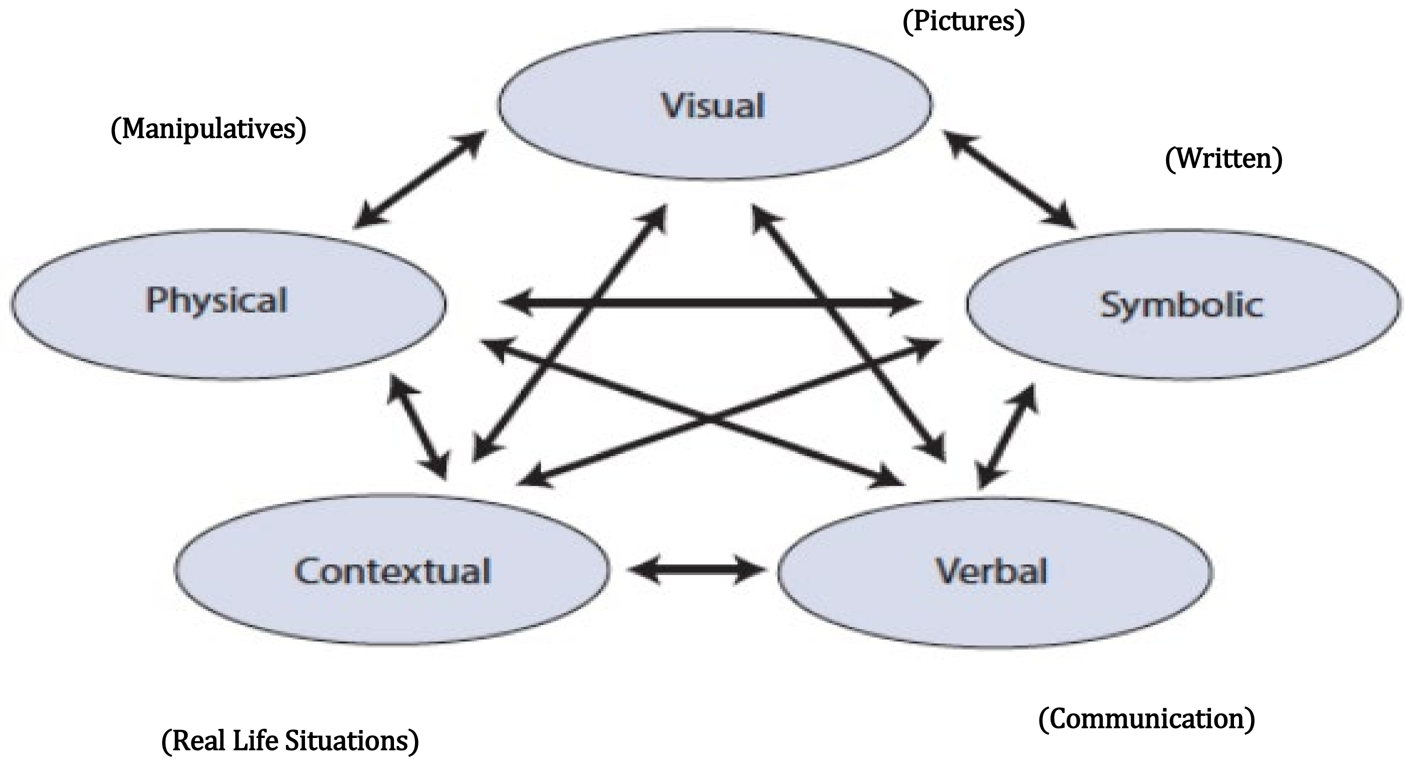
- Students believe that larger units will give the larger measure.
- Students should be given multiple opportunities to measure the same object with different measuring units.
- Students should notice it takes fewer yard sticks to measure the room than rulers or tiles.
- Students have trouble converting from meters to centimeters, have them write steps.

PARCC Assessment Evidence/Clarification Statements

	Evidence Statement	Clarification	Math Practices
4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 30/100=	i) Tasks do not have a context.	MP.7
4.NF.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	i) Measuring to the nearest mm or cm is equivalent to measuring on the number line.	MP.7
4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols > =, , or	i) Tasks have "thin context" or no context. ii) Justifying conclusions is not assessed here. iii) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.	MP.5, MP.7
4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12) , (2, 24) , and (3, 36) ,…	None	MP.5, MP.8
4.MD.4.1	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8).	None	MP.5
4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points		MP 2

	where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.		
4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. -		MP 2, 5
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. -		MP 1,7
4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two dimensional figures.		MP 5
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	• A trapezoid is defined as “A quadrilateral with at least one pair of parallel sides.” • Tasks may include terminology: equilateral, isosceles, scalene, acute, right, and obtuse.	MP 7

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

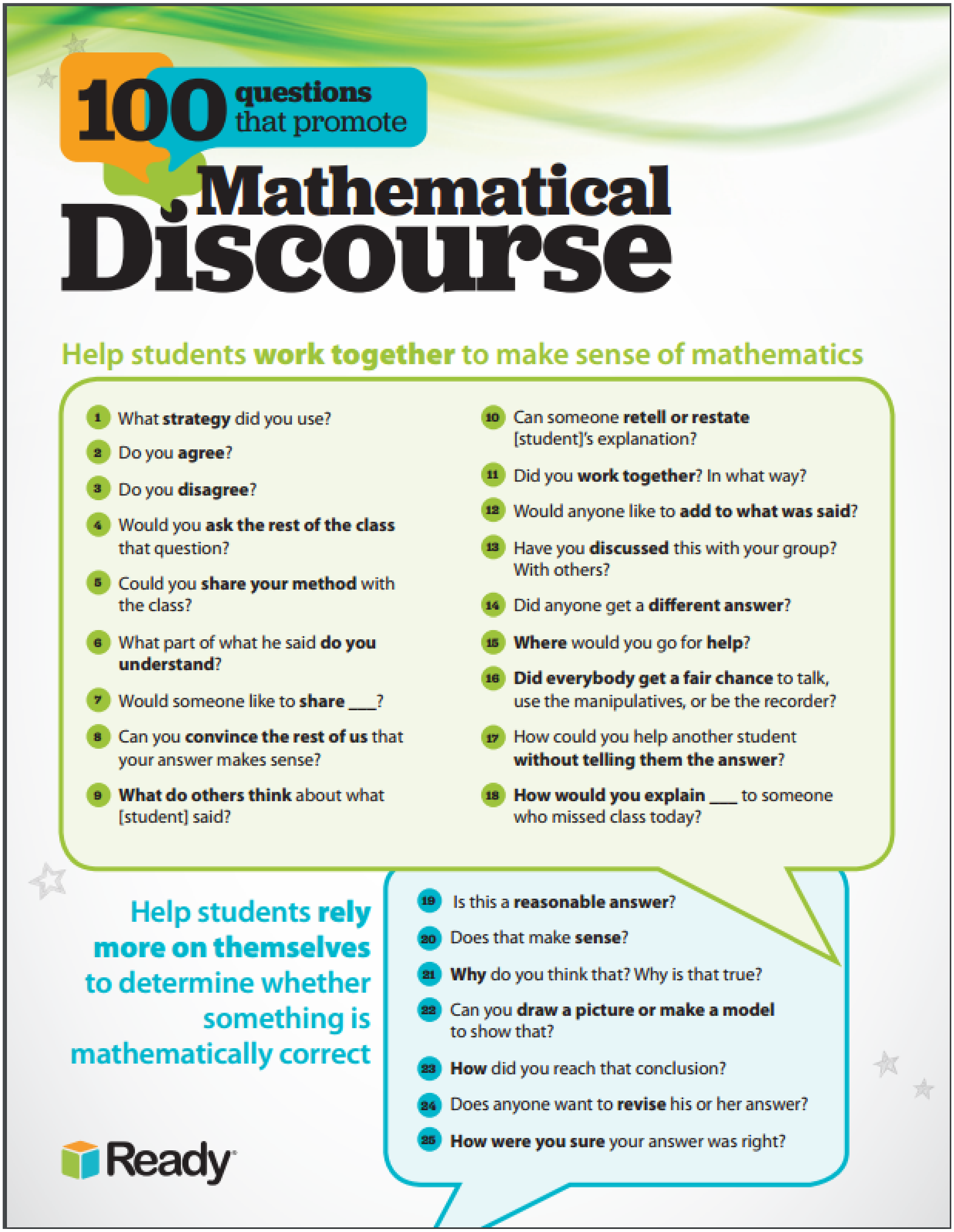
While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

<u>**Teacher Questioning:**</u>

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts**. What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74. What is the **relationship** between ___ and ___?
75. Have we ever solved a problem **like this before**?
76. What uses of mathematics did you find in the **newspaper** last night?
77. What is the **same**?
78. What is **different**?
79. Did you use skills or build on concepts that were **not necessarily mathematical**?
80. Which **skills or concepts** did you use?
81. What **ideas** have we explored before that were useful in solving this problem?
82. Is there a **pattern**?
83. **Where else** would this strategy be useful?
84. How does this **relate** to ___?
85. Is there a **general rule**?
86. Is there a **real-life situation** where this could be used?
87. How would your method work with **other problems**?
88. What other problem does this seem to **lead to**?

Help students **persevere**

89. Have you tried making a **guess**?
90. **What else** have you tried?
91. Would **another method** work as well or better?
92. Is there **another way** to draw, explain, or say that?
93. Give me another **related problem**. Is there an easier problem?
94. How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95. What was **one thing you learned** (or two, or more)?
96. Did you **notice any patterns**? If so, describe them.
97. What **mathematics topics** were used in this investigation?
98. What were the **mathematical ideas** in this problem?
99. What is mathematically **different about these two situations**?
100. What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)

3.NBT.A.2: Add/subtract within 1000

4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm

5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

Connections to the Mathematical Practices

1	**Make sense of problems and persevere in solving them**
	Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively**
	Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others**
	In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

4	**Model with mathematics**	
	Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.	
5	**Use appropriate tools strategically**	
	Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.	
6	**Attend to precision**	
	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.	
7	**Look for and make use of structure**	
	In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.	
8	**Look for and express regularity in repeated reasoning**	
	Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.	

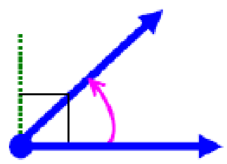
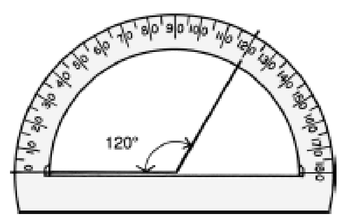
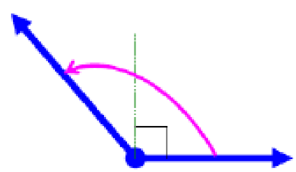
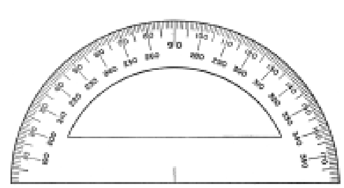
Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discourse	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

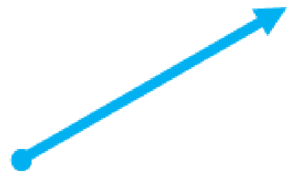
Visual Vocabulary

Visual Definition
The terms below are for teacher reference only and are not to be memorized by students. Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

Chapter 9

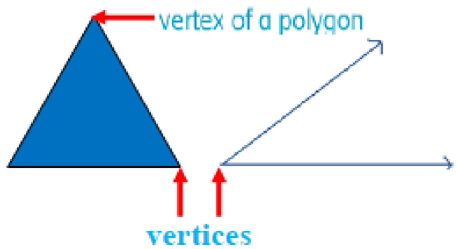
acute angle		An angle with a measure less than 90°.
degree (angle measure)		A unit for measuring angles. Based on dividing one complete circle into 360 equal parts.
obtuse angle		An angle with a measure greater than 90° but less than 180°.
protractor		A tool used to measure and draw angles.

ray



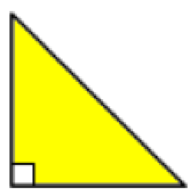
A part of a line that has one endpoint and goes on forever in one direction.

vertex



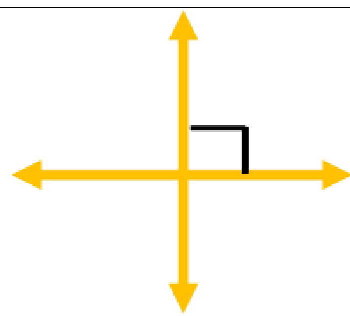
The point at which two line segments, lines, or rays meet to form an angle.

right triangle



A triangle that has one 90° angle.

perpendicular lines



Two intersecting lines that form right angles.

parallel lines

Lines that are always the same distance apart. They do not intersect.

Chapter 12

kilometer (km)

A kilometer (km) is about the length of 4 city blocks.

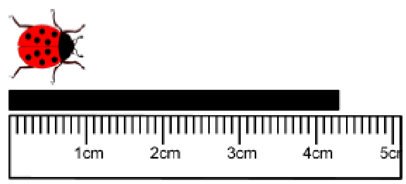
A metric unit of length equal to 1000 meters.

gram (g)

The mass of a paperclip is about 1 gram.

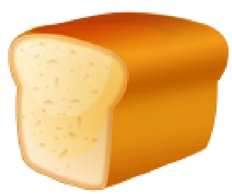
The standard unit of mass in the metric system. 1,000 grams = 1 kilogram

centimeter (cm)



A metric unit of length equal to 0.01 of a meter.

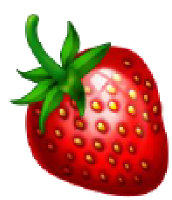
pound (lb)



A loaf of bread weighs *about* 1 pound.

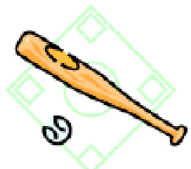
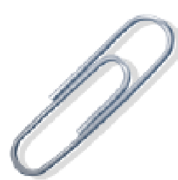
A customary unit of weight.
1 pound = 16 ounces.

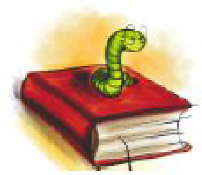
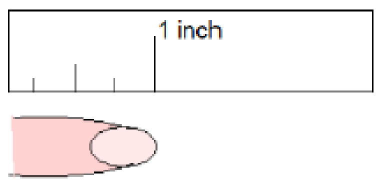
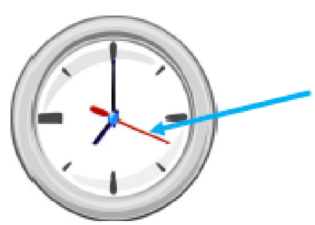
ounce (oz)

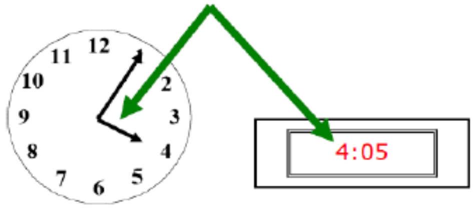
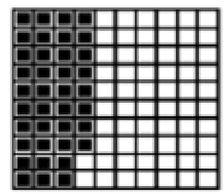
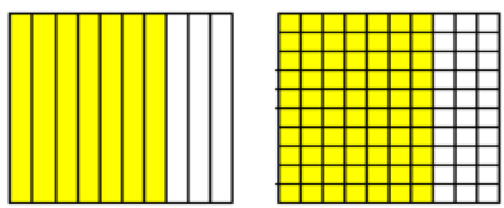


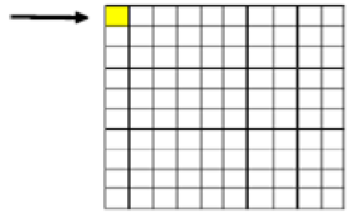
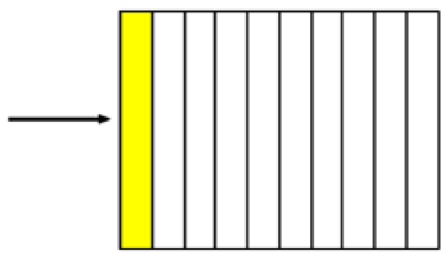
A strawberry weighs *about* 1 ounce.

A customary unit of weight equal to one sixteenth of a pound. 16 ounces = 1 pound.

mile	 **Two times around the average roller coaster is *about* 1 mile.**	A customary unit of length. 1 mile = 5,280 feet
meter (m)	 **A baseball bat is *about* 1 meter long.**	A standard unit of length in the metric system.
yard (yd)	 A door is *about* 1 yard wide.	A customary unit of length. 1 yard = 3 feet or 36 inches.
foot (ft)	12 inches = 1 foot 	A customary unit of length. 1 foot = 12 inches.
gram (g)	**The mass of a paperclip is about 1 gram.** 	The standard unit of mass in the metric system. 1,000 grams = 1 kilogram

kilogram (kg)	 Math book **About 2 ½ pounds**	A metric unit of mass equal to 1000 grams.
liter (L)	large bottle of soda or bottle of water  1,000 mL = 1 L	The basic unit of capacity in the metric system. 1 liter = 1,000 milliliters.
inch (in)		A customary unit of length. 12 inches = 1 foot.
minute (min)		One sixtieth of an hour or 60 seconds.
second (sec) **(unit of time)**	 **60 seconds = 1 minute**	One sixtieth of a minute. There are 60 seconds in a minute.

hour (hr)	 A unit of time. 1 hour = 60 minutes. 24 hours = 1 day.
decimal	\$29.45 53.0 0.02
decimal fraction	 $0.38 = \frac{38}{100}$
decimal point	\$1.55 3.2 decimal point
equivalent decimals	 0.7 = 0.70
greater than	 5 > 3

hundredth		One of the equal parts when a whole is divided into 100 equal parts.
hundredths	4.38	In the decimal numeration system, hundredths is the name of the next place to the right of tenths.
order	$\frac{2}{8}$ $\frac{2}{6}$ $\frac{2}{4}$ **In order from least to greatest.**	
tenth		One of the equal parts when a whole is divided into 10 equal parts.
tenths	4.3	In the decimal numeration, tenths is the name of the place to the right of the decimal point.

Assessment Framework

Unit 3 Assessment/Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math Module 6: Decimal Fractions (TOPICS C,D,E)			
Chapter 12			
Optional End-Module Assessment	4.NF.5-7	1 Block	Individual
Optional Chapter Test 12	4.MD.1	1/2 Block	Individual
Authentic Assessment: Toy Car Distances	4.MD.1	1/2 Block	Individual
Chapter 9	Fractions Equivalence, Ordering, and Operations	(TOPICS B, C, D)	
Optional Chapter Test 9	4.G.1 4.MD.5-7	1/2 Block	Individual
Authentic Assessment: Matthew and Nick's Circles	4.MD.5	1/2 Block	Individual
Chapter 10			
Optional Chapter Test 10	4.G.1-2	1/2 Block	Individual
Grade 4 Interim Assessment 3 (i-Ready)	4.MD.1, 5,6,7 4.G.1-2 4.NF. 5,6,7	1-2 Blocks	Individual

	PLD	**Genesis Conversion**
Rubric Scoring	PLD 5	100
	PLD 4	89
	PLD 3	79
	PLD 2	69
	PLD 1	59

Hannah's class was conducting a science experiment using toy cars. They would roll a car down a ramp and measure how far the car traveled. When Hannah filled out her recording sheet about her car, she wrote the following:

The car went a distance of 4 long (or 48 long).

Hannah's answer did not include the units she used. She used two of the units shown in the UNIT BANK. Fill in the blanks to show the units that she used when measuring the distance her car traveled.

UNIT BANK
millimeters inches yards meters feet centimeters

The car went a distance of 4 ________ (or 48 ________).

Hannah's classmate Caleb conducted the same car experiment, and he also forgot to write the units with his data. Hannah filled in the units that she thought he used.

The car went 3 **centimeters** (or 300 **meters**).

Explain why the units Hannah chose make the statement unreasonable.

____

____

____

____

Toy Car Distances

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

• Students may need to do calculations on paper, either to solve or to check their work. Encourage the students to use any space on the paper to show their thinking. Some students may require more space than the paper provides or may need the lines of notebook paper to structure their work. You may choose to give those students, or all students, extra paper on which they can do their calculations.

• A student should fill in the first pair of blanks with the words “feet” and “inches”, respectively. The student’s explanation for the final part of the task should indicate an understanding that centimeters are smaller than meters and that if a given length is measured in both centimeters and meters, that the number used for centimeters will be larger than the number used for meters. The level of specifics in the answers can help distinguish “substantial” from “full” accomplishment.

• As indicated in the rubric, students may make minor errors that do not relate to the target concept (i.e., not labeling numbers), but if the work shows a complete understanding of the relationship between units, they can still be rated as showing “full accomplishment”.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas. May have minor errors that do not impact the mathematics.	Student could work to full accomplishment with minimal feedback from teacher. Errors are minor. Teacher is confident that understanding is adequate to accomplish the objective with minimal assistance.	Part of the task is accomplished, but there is lack of evidence of understanding or evidence of not understanding. Further teaching is required.	The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little or no success. Further teaching is required.	The student shows no work or justification

Matthew and Nick were investigating angles and circles, drawing circles and creating angles inside of their circles.

Matthew drew a small circle and divided it into six equal sections. He measured the angles of each section and found that they were all 60º.

Nick decided to draw a circle that was larger than Matthew's circle. He divided his circle into six equal sections and measured the angles of each section. He expected them to be larger than 60º, but they all measured 60º.

The resource sheet **Circles and Angles** shows the work that Matthew and Nick did.

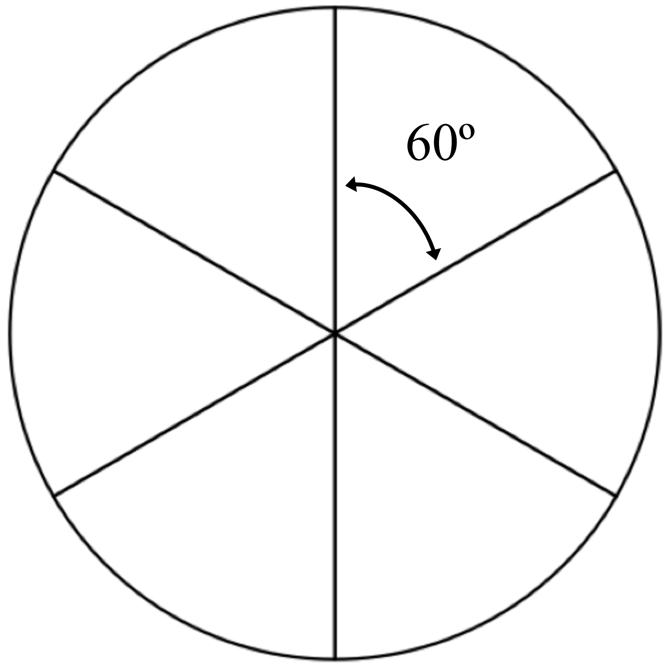
Why might Nick have thought the sections of his circle would have a larger angle measurement than the sections in Matthew's circle?

Why do the sections in Nick's circle and the sections in Matthew's circle have the same angle measurement?

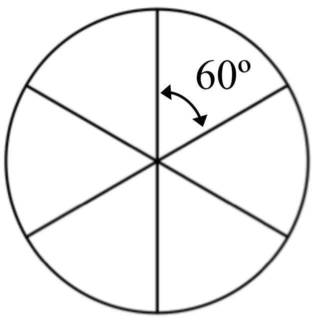
Circles and Angles

Resource Sheet

Nick's circle



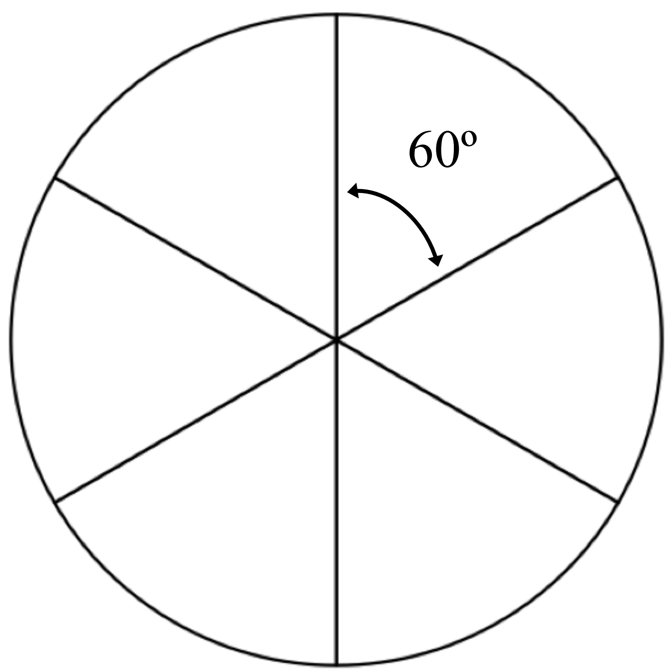
Matthew's circle



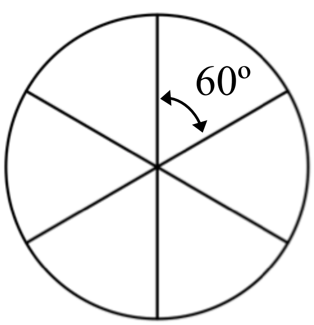
Circles and Angles

Resource Sheet

Nick's circle



Matthew's circle



4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

SOLUTION:

See below

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on explanations/reasoning using the: • parts of an angle and define what an angle is • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes an **efficient** and logical progression of steps.	Clearly constructs and communicates a complete response based on explanations/ reasoning using the: • parts of an angle and define what an angle is. • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes a **logical** progression of steps	Constructs and communicates a complete response based on explanations/ reasoning using the: • parts of an angle and define what an angle is. • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Constructs and communicates an incomplete response based on explanations/ reasoning using the: • parts of an angle and define what an angle is. • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes an **incomplete or Illogical** progression of steps.	The student shows no work or justification

Additional Assessment Resources

Literature

Literature Fractions and Decimals Made Easy, by Rebecca Wingard-Nelson Fun Food Word Problems Starring Fractions, by Rebecca Wingard-Nelson
The Hershey's Milk Chocolate Fractions Book, by Jerry Pallotta
Jump, Kangaroo, Jump!, by Stuart J. Murphy Polar Bear
Math: Learning About Fractions from Klondike and Snow, by Ann Whitehead Nagda
The Wishing Club: A Story About Fractions, by Donna Jo Napoli
Working With Fractions, by David A. Adler

Project Ideas:
Doubling A Recipe
Mozaic Art

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .